

LIE ALGEBRA AND LIE GROUP BUNDLES: A SURVEY

B. S. KIRANAGI

University of Mysore, India

Abstract: Serre posed the question: For every given Lie algebra bundle ξ , does there exist a Hausdorff Lie group bundle whose Lie algebra bundle is isomorphic to a given Lie algebra bundle ξ ?

In 1966, A. Douady and M. Lazard constructed a Lie group bundle Γ (not necessarily Hausdorff) for a given Lie algebra bundle ξ such that the Lie algebra bundle of a Lie group bundle Γ is isomorphic to a given Lie algebra bundle ξ in their remarkable paper [3]. They had left the following problem open in an analytic case. That is, when all the spaces of a bundle are analytic manifolds and the corresponding morphism are analytic.

Problem: Given a Lie algebra bundle $\xi = (\xi, p, X)$ over a Hausdorff space X . Does there exist for each s in X , an open set S of s in X and a Hausdorff Lie group bundle over S whose Lie algebra bundle is isomorphic to Lie algebra bundle $\xi_S = p^{-1}(S)$. They also suggested tackling this question from the point of view of algebraic geometry [3, p. 151].

Here we answer this open problem partially by proving a fundamental result in algebraic geometry: The real orbit of a real point is open in the real part of its complex orbit. Further, we also cover the cohomology and representation theory of Lie algebra bundles and the relation between other bundles.

References

- [1] B. S. Kiranagi, Lie algebra bundles, Bull. Sci. Math. (2) 102 (1978), 57-62.
- [2] B. S. Kiranagi, Semi simple Lie algebra bundles, Bull. Math de la Sci. Math de la R. S. de Roumaine 27(75) (1983), 253-257.
- [3] A. Douady and M. Lazard, Espaces fibres en algebre de Lie et en groupes, Invent. Math. 1 (1966), 133-151.
- [4] B. S. Kiranagi and G. Prema, On complete reducibility of module bundles, Bull. Austral. Math. Soc. 28 (1983), 401-409.
- [5] B. S. Kiranagi, Lie algebra bundles and Lie rings, Proc. Nat. Acad. Sci. India 54(A) (1984), 38-44.

- [6] B. S. Kiranagi and G. Prema, Cohomology of Lie algebra bundles and its applications, *Indian J. Pure Appl. Math.* 16 (1985), 731-735.
- [7] B. S. Kiranagi and G. Prema, Lie algebra bundles defined by Jordan algebra bundles, *Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.)* 31(79) (1987), 255-264.
- [8] B. S. Kiranagi and G. Prema, A decomposition theorem for Lie algebra bundles, *Comm. Algebra* 18(6) (1990), 1869-1877.
- [9] B. S. Kiranagi, G. Prema and C. Chidambara, Rigidity theorem for Lie algebra bundles, *Comm. Algebra* 20(6) (1992), 1549-1556.
- [10] B. S. Kiranagi and R. Rajendra, Revisiting Hochschild cohomology for algebra bundles, *J. Algebra Appl.* 7(6) (2008), 685-715.