

A SURVEY ON THE NEUTRAL GEOMETRY IN DIMENSION FOUR

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Abstract: In Relativity theory, space-time manifolds are four-dimensional pseudo-Riemannian manifolds of Lorentz signature, i.e., $(+++ -)$. The existence condition of such Lorentz metrics on manifolds are well-known, as that a manifold must admit a nonsingular vector field. We are interested in a similar problem of the existence of metrics of various signatures. We then focus our attention to the metrics of signature $(++ - -)$, usually called *neutral metrics*, which are the lowest dimensional example of indefinite metrics not of Lorentz type. My talk is concerned with the existence theorem of neutral metrics in dimension four. This issue is closely related to various topics in differential geometry, differential topology, algebraic geometry and so on. It should be noted that the basic parts of the theorem are from the works of Steenrod (1951), Hirzebruch and Hopf (1958), and that a breakthrough to the theorem is provided by Donaldson's work (1983), which was celebrated as the Fields Prize in 1986. My talk will cover various interesting topics related to the neutral geometry, and I will end my talk with a brief introduction of my recent works of spinor approach to neutral geometry which are joint works with Peter R. Law, a Ph.D. student of Professor Sir Roger Penrose.