ON THE SEMIRING \( \left( \begin{array}{cc} R & \Gamma \\ S & L \end{array} \right) \)

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Abstract: H. S. Vandiver introduced the notion of semiring as a generalization of ring in the year 1934. A ring \( R \) becomes a semiring when \( (R, +) \) is a semigroup instead of a group. In 1964, N. Nobusawa, to provide algebraic home to \( \text{Hom}(A, B), \text{Hom}(B, A) \) and to \( M_{m \times n}(R), M_{n \times m}(R) \), where \( A \) and \( B \) are additive abelian groups and \( R \) is a ring, introduced the notion of an algebraic structure what is known as \( \Gamma \)-ring. \( \Gamma \)-ring not only generalizes the notion of ring but also the notion of ternary ring. In 1981, M. K. Sen further generalized the notion of \( \Gamma \)-ring and introduced the notion of \( \Gamma \)-semigroup. Taking impetus from M. K. Sen’s work on \( \Gamma \)-semigroup M. M. K. Rao introduced the notion of a \( \Gamma \)-semiring in 1995. It turns out to be a generalization of semiring as well as of \( \Gamma \)-ring. In 2002 the present author and T. K. Dutta introduced the notion of operator semirings of a \( \Gamma \)-semiring and developed the theory of \( \Gamma \)-semiring to a considerable extent. In 2008 the present author and B. C. Saha introduced the notion of a both sided \( \Gamma \)-semiring, i.e., Nobusawa \( \Gamma \)-semiring. To every Nobusawa \( \Gamma \)-semiring one can associate a matrix like semiring \( \left( \begin{array}{cc} R & \Gamma \\ S & L \end{array} \right) \), where \( R \) and \( L \) are respectively the right and left operator semirings of \( S \). Though the concept of a semiring generalizes that of a ring the ideal properties of semiring sometimes differ from the properties of ring ideals. In order to amend this gap, the concept of \( k \)-ideals and \( h \)-ideals in a semiring were introduced by D. R. LaTorre in 1965. By using the notion of \( k \)-ideal of a semiring Olson et al. introduced the notions of pre-prime and pre-semiprime ideals. The present author and B. C. Saha showed that like \( k \)-ideals, \( h \)-ideals could also be used to define new types of ideals called \( h \)-prime and \( h \)-semiprime ideals in semirings. These notions were then transferred to \( \Gamma \)-semirings. In this talk, we investigate the properties of \( k \)-prime, \( k \)-semiprime, \( h \)-prime and \( h \)-semiprime ideals in the matrix semiring \( S_2 = \left( \begin{array}{cc} R & \Gamma \\ S & L \end{array} \right) \).

Besides this we show that the semiring \( S_2 \) provides an example of Morita context for semiring introduced by Dutta and Das. This in other words means that corresponding to every Nobusawa \( \Gamma \)-semiring there exists a Morita context for semirings. To conclude the talk we discuss the reverse question namely: Does there exist a Nobusawa \( \Gamma \)-semiring \( S \) corresponding to a Morita context of semirings such that the Morita context induced by \( S \) is the given one?