# CONTINUOUS WAVELET TRANSFORM ON A ONE-SHEETED HYPERBOLOID

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**Abstract:** The one-sheeted hyperboloid $H^{1,1}$ with Cartesian equation $x_0^2 - x_1^2 - x_2^2 = -1$ may be parameterized as

$$x = (\sinh \chi,\ \cosh \chi \cos \varphi,\ \cosh \chi \sin \varphi),$$

where $x \in \mathcal{R}$, $0 \le \varphi < 2\pi$. The motions on $H^{1,1}$ are of two types: (i) rotations: $x(\chi, \varphi) \mapsto (\chi, \varphi + \varphi_0)$; and (ii) displacements: $x(\chi, \varphi) \mapsto (\chi + \chi_0, \varphi)$. They constitute the group $SO_0(1, 2)$. To define dilation on $H^{1,1}$, project it onto the cone $\mathcal{C} = \{\xi = (\xi_0, \xi_1, \xi_2) \in \mathbb{R}^3 : \xi_0^2 - \xi_1^2 - \xi_2^2 = 0\}$, with dilation $\xi \mapsto a\xi$. In polar coordinates, the dilation operator acts on a point $x(\chi, \varphi)$ by $D_a(\chi, \varphi) = (\chi_a, \varphi)$, with $\sinh \chi_a = a \sinh \chi$.

For all test functions $f$ on $H^{1,1}$, introduce the following pair of transforms $\hat{f}_{\pm}(\nu, \xi) = \langle f(x), \varepsilon^{\pm}_{\nu,\xi}(x)\rangle$, where $\nu \in \mathbb{R}^+$, $\xi$ varies on the half cone $\mathcal{C}^+ = \{\xi \in \mathcal{C} : \xi_0 > 0\}$ and the kernels $\varepsilon^{\pm}_{\nu,\xi}(x)$ are *hyperbolic plane waves*. This transformation is called the *Fourier-Helgason transform*.

Let $\psi \in L^2(H^{1,1})$ be a symmetric and rotation invariant function, $a \mapsto \alpha(a)$ be a positive function on $\mathbb{R}^+_*$. Then we say that $\psi$ is *admissible* if there exist constants $m$ and $M$ such that $0 < m \le \mathcal{A}_\psi(\nu) := \int_0^\infty |(\widehat{\psi_a})_\pm(\nu)|^2 \alpha(a)\, da \le M < \infty$. Given an admissible $H^{1,1}$-wavelet $\psi$, the $H^{1,1}$-*continuous wavelet transform* of $f \in L^2(H^{1,1})$ is $W_f(a, g) := \langle \psi_{a,g}, f\rangle = \int_{H^{1,1}} \overline{\psi_a(g^{-1}x)} f(x)\, d\mu(x)$, $g \in SO_0(1, 2)$, $a > 0$, where $\psi_a(x) = \lambda(a, x)^{1/2} \psi(D_{1/a}x)$ such that $\lambda(a, x)$ is the Radon-

Nikodym derivative. We show that a symmetric and rotation invariant function $\psi \in L^2(H^{1,1})$ is admissible if $\alpha(a)da$ is a homogeneous measure of the form $\alpha^{-\beta}da$ with $\beta > 3,$ and the following zero-mean condition is satisfied:

$$\int_{H^{1,1}} \psi(\chi, \varphi)\, d\mu(\chi, \varphi) = 0.$$

**References**


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[2] J. Bros and U. Moschella, Fourier analysis and holomorphic decomposition on the one-sheeted hyperboloid, 2003. [arXiv: math-ph/0311052v1].